VOLUME 77 SEPARATE No. D-11

Rochester Public Library

PROCEEDINGS

AMERICAN SOCIETY OF CIVIL ENGINEERS

March, 1951

DISCUSSION OF LONG-TERM STORAGE CAPACITY OF RESERVOIRS

(*Published in April, 1950*)

By Ven Te Chow, Henri Milleret, Louis M. Laushey, and H. E. Hurst

HYDRAULICS DIVISION

Copyright 1951 by the AMERICAN SOCIETY OF CIVIL ENGINEERS
Printed in the United States of America

Headquarters of the Society
33 W. 39th St.
New York 18, N.Y.

PRICE $0.50 PER COPY

V6206

The Society is not responsible for any statement made or opinion expressed in its publications

---

HINTS TO AUTHORS

Those who are planning papers for submission to PROCEEDINGS *will expedite Committee action measurably by first studying the standard instructions as to style, content, and format. For reprints, address the Manager, Technical Publications, 33 W. 39th St., New York 18, N. Y.*

Published at Prince and Lemon Streets, Lancaster, Pa., by the American Society of Civil Engineers. Editorial and General Offices at 33 West Thirty-ninth Street, New York 18, N. Y. Reprints from this publication may be made on condition that the full title of paper, name of author, page reference, and date of publication by the Society are given.

DISCUSSION

VEN TE CHOW.[13]—The solution of many hydraulic engineering problems requiring the use of past hydrologic data is often handicapped by the short lengths of the periods of record. Mr. Hurst has contributed a desirable and new idea in proposing that the data of long-period records covering a large number of natural events can be adapted to supply the deficiency in hydrologic data, although great accuracy cannot be expected from the application of the very general results, thus obtained, to the individual cases. In order to perfect the idea many raw data should be assembled and processed. The amount of time and labor consumed in the preparation of the paper must have been truly tremendous.

The statement following Eq. 12—

> "In each main group of observations a line drawn through the point log $(R/\sigma) = 0$, log $N = 0.30$ $(N = 2)$, and the center of gravity of all the observations is a good fit."

—seems open to some question. In each main group of observations, the writer has tried to fit a straight line by the method of least squares, with the results shown in Table 14. There is no evidence that all the equations will be

TABLE 14.—EQUATIONS OF BEST FIT

Phenomena	Number of groups	Equations; R/σ =
River level, discharges, etc.	14	$0.30\ N^{0.88}$
Rainfall	6	$0.43\ N^{0.79}$
Temperature and pressure	7	$0.37\ N^{0.82}$
Annual growth of tree rings	6	$0.36\ N^{0.86}$
Varves (Lake Saki in the Crimea)	7	$0.30\ N^{0.94}$
Varves (Tamiskaming, Ont., Canada, and Moen, Norway)	6	$0.43\ N^{0.77}$
Sunspot numbers and wheat prices	3	$4.04\ N^{0.29}$

satisfied by the condition log $(R/\sigma) = 0$, log $N = 0.30$. The equation derived for sunspot numbers and wheat prices cannot be trusted because the number of points to be fitted (three points) is too small. The parameters in other equations can be found to vary within reasonable ranges—from $a = 0.30$ to $a = 0.43$ and from $b = 0.77$ to $b = 0.94$.

A line designed to fit the main groups of all observations by the method of least squares was also found. It is expressed by

$$\frac{R}{\sigma} = 0.31\ N^{0.87} \qquad (17)$$

Eq. 17 is plotted to a log-log scale in Fig. 11, and appears to fit better than Mr. Hurst's line. The writer believes that a more satisfactory result would be

NOTE.—This paper by H. E. Hurst was published in April, 1950. The numbering of footnotes, equations, tables, and illustrations in this Separate is a continuation of the consecutive numbering used in the original paper.

[13] Hydr. Eng. Laboratory, Dept. of Civ. Eng., Univ. of Illinois, Urbana, Ill.

obtained by fitting the line directly to all individual cases instead of to their main groups. In the latter method the weight of individual observations has been disregarded.

For $N = 2$, the ratio R/σ has the exact value $R/\sigma = 1$; and, for $N = 1$, $R = 0$ (or for $\log N = 0$, $\log (R/\sigma) = -\infty$). If the first condition ($N = 2$) is to be emphasized, the other condition ($N = 1$) must also be taken into

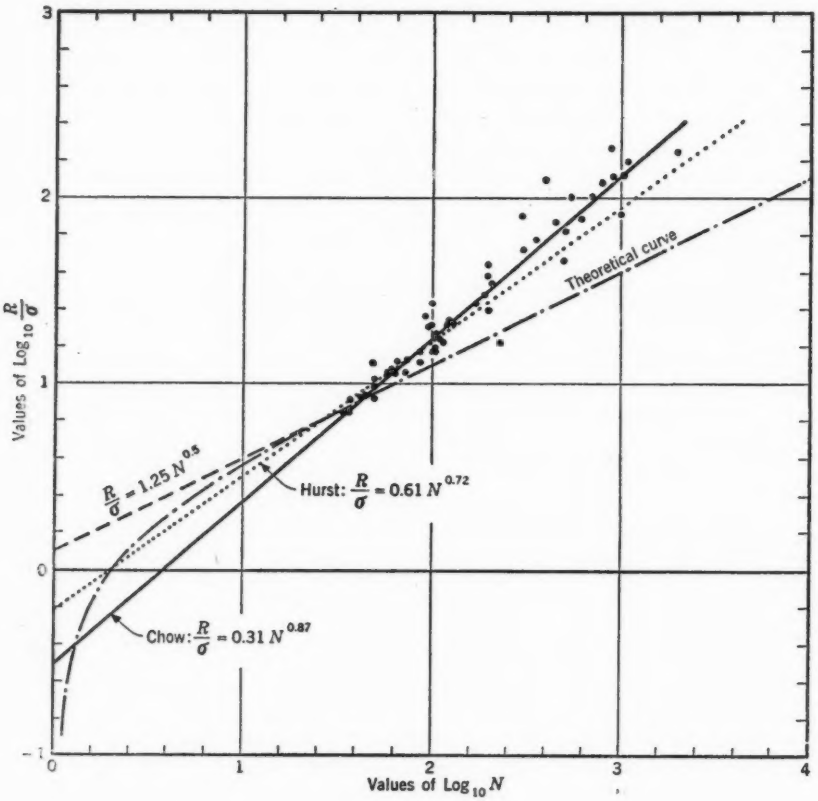

FIG. 11

consideration. Consequently, the true theoretical plot for $\log (R/\sigma)$ against $\log N$, if considered as a continuous function, is not a straight line, but a curve, as shown in Fig. 11. This curve is asymptotic to the $\log (R/\sigma)$-axis and tends (as N increases) to approximate the line defined by Eq. 10.

Also, in Section 8, the author states:

> "In general, R has not been computed for periods of less than 30 years because, for very short periods, it has little practical value although it may have some formal value."

This statement is correct also because, for very short periods, the standard deviation computed from the observed data is subject to great variation, and, hence, a reliable value of R/σ cannot be obtained easily. Formulas, such as Eqs. 10, 13, or 17, may be accepted as valid only for large values of N, and, therefore, it is difficult to reason that the linear relationship between log (R/σ) and log N must hold, down to the point $N = 2$.

HENRI MILLERET.[14]—The paper by Mr. Hurst is of special interest to the writer since he himself is concerned with a similar problem. The general conclusions in Section 4 are reasonable, providing for both economy and safety.

As the author has stated, there are objections to using the Gaussian normal probability law; actual distributions of natural phenomena always have a certain skew. Nevertheless, he uses this law as a basis for his researches and computations.

It is now well established that a formula having three coefficients fits the observed data far better than one having two coefficients (such as the Gaussian law). In this connection, the logarithmic transformation, known as Gibrat's law is still used successfully.[15] It fits observed data of such different events as river discharge, flood peaks, rainfall, personal income, and population.[16]

The fact that the Gaussian law was found to check quite well with the many various phenomena reported by Mr. Hurst is understandable since, in all the cases, the range of observations is relatively small. For instance, in Table 1 the annual discharge, in milliards, ranges from 13 to 48 (ratio 3.7). It is possible that for such a small ratio, a Gaussian curve will always fit. It must be emphasized, for instance, that rivers in North Africa have much larger ratios. Mr. Hurst himself stresses the fact that his formulas are valuable only within the range of observations.

Nevertheless, the use of a "skew formula" would yield a safer theoretical basis for research, and would allow the results to be extended to more "erratic" cases. It should be added that hydraulic engineers still favor the formulas with three constants.

Even when the Gaussian formulas are used, an actual mean is not recommended in the computation. The statistical mean is not known in advance, but can be decided. The best values are available by using probability paper, and plotting observed events against Gaussian frequencies. The ordinary equation of the line of best fit then yields the statistical "true" value of the mean and, at the same time, of the standard deviation, which is not possible by ordinary arithmetic.

Technical literature on this subject is very scarce, and papers on researches such as Mr. Hurst has reported will be always welcomed by hydraulic engineers.

[14] Engr., École Nationale des Ponts & Chaussées Société des Grands Equipements Marocains, Agadir, French Morocco.

[15] "Rainfall and Runoff," by Edgar Eugene Foster, Macmillan Co., New York, N. Y., 1948.

[16] "Aménagement hydroélectrique des cours d'eau: statistique mathématique et calcul des probabilités," by R. Gibrat, *Revue Générale de l'Électricité*, tome XXXII, Nos. 15 and 16, Paris, 1932.

LOUIS M. LAUSHEY,[17] JUN. ASCE.—It has been shown that reservoir storage capacity can be determined by statistical relationships based on many observations of several different natural phenomena. The theory is based on a normal distribution from the mean and a random sequence of occurrence of values. Because of the long-term cyclic effects of nature, observed data do not check the theoretical equation, and corrections have been determined. The author must be admired for his thoroughness and sympathized with for the many computations involved. One simplification will be proposed.

The measure of dispersion, σ, can also be expressed by the mean absolute deviation, m. The variable, σ, could be replaced in Eq. 12 by a function of m to provide an alternate form that may have advantages. The choice of variable is naturally a matter of personal opinion, and the standard deviation has certain significance, but the following points are offered to support the possible change to the mean absolute deviation, m:

1. Computations are simplified considerably by the omission of the squares of deviations;
2. Work is completely self-checking;
3. Most books on statistics show the following relationship between σ and m for a normal curve:

$$\frac{m}{\sigma} = \sqrt{\frac{2}{\pi}} = 0.798 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (18)$$

or, therefore,

$$\frac{R}{m} = 1.253 \left(\frac{N}{2}\right)^{K} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (19)$$

4. The constant 0.798 is stable and varies little for curves deviaitng from a normal curve;
5. The value m is just as rational a choice since R is directly a function of the deviations, not of the squares; and
6. Computations of turbulence fluctuations to be presented indicate that m should be a satisfactory variable.

One more natural phenomenon can be added to the list already studied. Measurements were made of the variations in the squares of the velocity in a turbulent stream of water. The distribution of the variations from the average was symmetrical, but slightly flatter than a normal curve. It is an interesting comparison because 300 points were recorded in just one minute, not hundreds of years. All computations were made from the same 300 points to study the changes in variables within one representative group of points. Hence, the exact value of K was not expected for the turbulence variations since it may vary from point to point or with time.

Table 15 shows the results of ten groups of 30 points, five groups of 60 points two groups of 150 points, and one group of 300 points—all recorded consecutively at intervals of one fifth of a second. The longitudinal component is in the direction of motion, and the transverse component is at right angles.

[17] Asst. Prof. of Civ. Eng., Carnegie Inst. of Technology, Pittsburgh, Pa.

The value of K' was computed from the mean absolute deviation by assuming $\sigma = 1.253\ m$. Values of K were computed from the standard deviation. Agreement is close, variations from the mean are about the same, and the assumed value of 0.798 checked. In Table 1, this ratio was calculated to be about 0.75 for the 43-year period. With more values, this ratio probably would approximate the theoretical within a small percentage.

The average value of K was about 0.66, which is less than 0.72 as determined by the author. Perhaps this is due to the small number of values used, but it is more likely that the tendency for cycles to occur during the short interval is not as great as found in long-term natural phenomena.

TABLE 15.—MEAN ABSOLUTE DEVIATIONS AND STANDARD DEVIATIONS OF TURBULENCE FLUCTUATIONS

m = mean absolute deviations of velocity-squared fluctuations;
K' = exponent of $\frac{N}{2}$ similar to K, but computed from $\sigma = \frac{m}{0.798}$; and
Time interval between each of 300 points is 0.2 sec.

Group	No. of points per group	No. of groups	Group Averages					
			$\frac{R}{\sigma}$	K	K'	m	σ	$\frac{m}{\sigma}$
(a) Longitudinal Component								
1	30	10	6.34	0.666 ±0.088	0.656 ±0.089	2.47	3.08	0.805
2	60	5	8.37	0.628 ±0.088	0.629 ±0.086	2.47	3.12	0.793
3	150	2	16.64	0.629 ±0.105	0.631 ±0.106	2.50	3.17	0.780
4	300	1	27.9	0.664	0.666	2.50	2.16	0.793
Average	....	....	....	0.647	0.646	....	....	0.793
(b) Transverse Component								
1	30	10	6.46	0.681 ±0.052	0.695 ±0.057	2.76	3.57	0.771
2	60	5	9.96	0.663 ±0.079	0.676 ±0.086	2.77	3.63	0.761
3	150	2	17.21	0.657 ±0.030	0.664 ±0.027	2.81	3.64	0.773
4	300	1	25.7	0.648	0.656	2.81	3.67	0.768
Average	....	....	....	0.662	0.673	....	....	0.768

There is no reason for the author to attempt to find a theoretical value of K for natural conditions, other than 0.5 for completely random occurrences, and no disturbing cyclic effects on the variables. Because of cyclic effects and different natural forces affecting each event, the values of K should be expected to vary slightly with each natural phenomenon.

H. E. HURST[18].—For their appreciative remarks and useful suggestions, the writer wishes to thank the discussers of this paper. Mr. Chow has raised the point that the lines in Fig. 4 are not the lines of best possible fit to the observed points; and he calculates other lines using the method of least squares to

[18] Scientific Consultant to the Ministry of Public Works, Egypt; formerly Director-Gen., Physical Dept., Cairo, Egypt.

determine two parameters instead of the single parameter chosen by the writer. Of course, the larger the number of parameters used, the better the fit will be. The reasons that determine the choice of a formula are either theoretical ones, or they are matters of practical convenience. The latter usually lead to choosing the simplest formula that fits the facts. In the second case there is no justification for complicating a formula with additional parameters if the resulting improvement of fit is not significant.

In the present case no theoretical formula has been discovered for natural phenomena, but it is a fact that all curves pass through the point corresponding to $N = 2$, $R/\sigma = 1$. Mr. Chow remarks that $N = 1$ must also be considered, but when $N = 1$, both R and σ are zero, and R/σ is indeterminate. Therefore, as the curve can only be determined for integral values of N, it stops at $N = 2$.

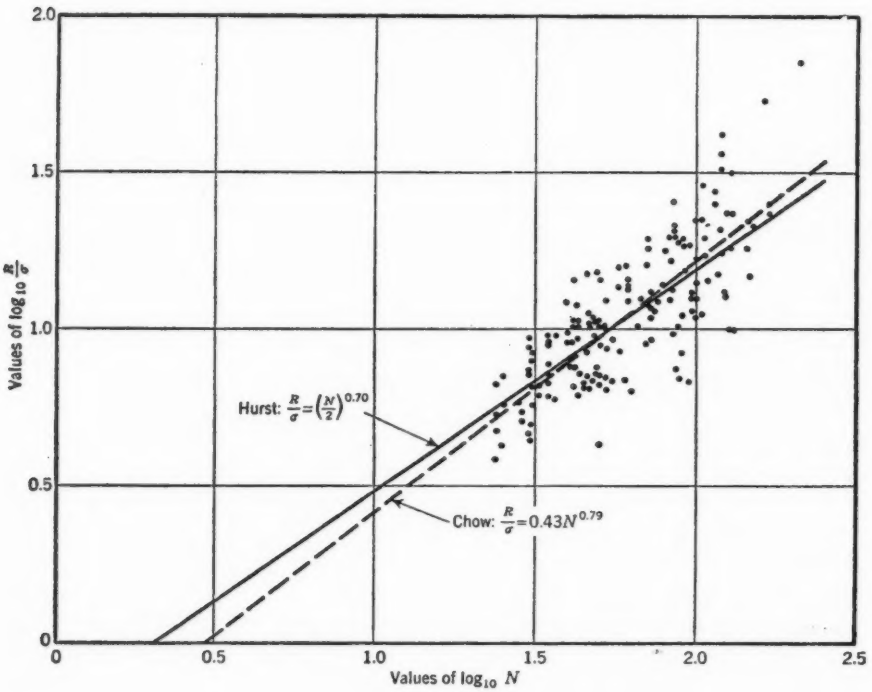

FIG. 12

The curves given by Eq. 12 clearly fit the observations very well. Therefore, Eq. 17 was compared with Eq. 12 to see if the difference was significant. Rainfall records (which are, perhaps, the most important group in practice) have been chosen, and the results of 168 calculations of R over periods ranging from 24 to 211 years have been plotted in Fig. 12. Lines representing Mr. Chow's Eq. 17 and the writer's Eq. 12 are also shown. There is a fairly large scatter among the individual points. In fact, compared with this scatter, the difference between the lines is not large enough to be significant. A rough confirmation of

this fact was obtained by taking two groups of values of log (R/σ), one for log $N = 1.48$ to 1.52, and the other for log $N = 1.98$ to 2.02, and finding the probable errors of their means without allowing for the small differences in log N. The differences between the lines were less than their probable errors and so were not significant. This conclusion applies to the other groups since the range of individual values of K is much the same in all of them. For the practical purpose of prediction and of deciding what value of R to use in a particular case, predictions are not likely to be extended to periods of less than 30 years, or more than 100 years, and any differences in formulas would be, therefore, entirely negligible.

In view of the foregoing and the remarks in the paper on Fig. 5 and Table 8, there seems to be no reason for adopting a more complicated formula than the one used in the paper.

Mr. Milleret's remarks about the use of a Gaussian curve or of a skew frequency curve illustrate a similar point. The writer agrees thoroughly about the use of probability paper, a valuable means of dealing with frequency distributions whether normal or skew. Probability paper is constantly used in the writer's practice.

Theoretical frequency distributions can be derived for purely chance events, such as tossing coins, and these are the only events to which they have been applied in the paper. It is not usually possible, however, to arrive at the frequency distribution of a natural event on theoretical grounds except, perhaps, as to its general form. In the paper R and σ have been calculated in every case from the actual observations, and they depend on no assumptions as to the form of frequency distributions.

The difference between R/σ for natural and for pure chance events is not due to skewness but to the fact that the distribution of natural events is not haphazard. It happens that irregular successions of high or low years tend to occur in any long series of years.

The 690 values of R and σ computed for the paper might be analyzed to discover any effect of skewness, but the labor would be very heavy and of doubtful value. The distribution of many natural events, such as those dealt with in the paper, approximates to the normal, and there are often insufficient observations to determine whether the distribution is normal or definitely skew. In these cases, much useless labor is saved in computation by keeping to the normal form if it is necessary to compute a frequency curve, or still more, by using probability paper on which to draw the curve. If some general equation is to be used for skew frequency curves, the question arises as to which of the various forms that have been proposed is to be used, and this problem again is more likely to be decided on grounds of practical convenience than on theoretical considerations.

Mr. Laushey's suggestion to replace the standard deviation by the mean absolute deviation is a good one and would save labor, unless the standard deviation were needed for some other reason. The values of R/σ calculated by him for turbulence fluctuations form a useful addition to those collected in the paper and have the advantage, as he states, that a long series can be produced in a short time. It would be interesting to note what results would come from,

say, two series of 1,000 each for comparison with the long-term records in the paper.

After discussion had been formally closed, a letter was received from S. P. Wing, M. ASCE, containing some valuable practical comments on the importance of fixing reservoir capacity by probability considerations, with which the writer is entirely in agreement. Mr. Wing refers to the economic difficulties that have arisen in the past through faulty determination of water supplies and of the required capacity to regulate them, and remarks "that the size of the reservoir to equalize the flow depends upon the risk one is willing to take of going dry; that it is a matter of probabilities." A method of assessing risks is given.

Mr. Wing believes that

> "* * * one conclusion of the author, if left without comment, might lead the reader to question the applicability of standard probability methods to these types of problems."

The conclusion referred to is that, although groups of high and low values do occur in purely chance events, their tendency to occur in natural events is greater. One example of this tendency is the records of the Nile River at Aswan and Roda in Egypt (Section 6 and Tables 2, 9, and 10). In commenting on this conclusion, Mr. Wing examines the long series of records of the Roda gage and, after removing the trend of the readings to rise as time goes on (which is usually thought to be due to silt deposition raising the bed), he finds that there are still discontinuities. He believes that

> "* * * a substantial part of the non-randomness found by the author in his numerical computations based on the Nile record arose from its non-homogeneous character."

It is quite true, as Mr. Wing states, that there are discontinuities in the Roda gage records, but they account for very little of the tendency for periods of high or low years to occur more frequently under those circumstances than in pure chance distributions. This can be seen by considering the record from 640 AD to 1440 AD which is more complete and homogeneous than the later part and is previous to the principal discontinuities. If this record is divided into periods of 100 years after removing trend, the mean value of K from the eight cases is 0.74, whereas if seven overlapping periods of 200 years are taken, the mean is 0.75. These shorter periods cannot be much affected by discontinuities.

Mr. Wing believes that thickness of tree rings and varves are also subject to criticism, although on different grounds. For example, tree rings may vary with changes of temperature or with disease, as well as with rainfall, and varves may vary with changes of availability of load or changes in culture or regime. This is true, but in the case of trees the causes, though complex, are natural. It is true also in the case of varves, since only in the case of Lake Saki (in the Crimea) can human agency have had any effect, and that only perhaps during the last tenth of the record.

Excluding Roda gage, tree rings and varves, and using river discharges, rainfall, temperature, and pressure, the mean value for K is 0.70, which is close to the mean for all the data (0.72). Assuming $N = 100$ years, the formula

derived from natural events, with $K = 0.70$ yields $R/\sigma = 15.5$, whereas the formula derived from tossing coins would give 12.0 (Table 6). It cannot be doubted, therefore, that in calculating storage for practical purposes, an extension of the ordinary methods based on the probability theory is required. This does not mean that the methods should be discarded, but they must be modified to meet such cases as the foregoing where the order of events as well as their magnitudes is important.

In conclusion it may be stated that work is continuing on the methods to be adopted in regulating long-term storage reservoirs. The results emphasize—should it be necessary to do so—the importance of allowing a reasonable factor of safety on the value (R) calculated for the storage.

AMERICAN SOCIETY OF CIVIL ENGINEERS

OFFICERS FOR 1951

PRESIDENT

GAIL A. HATHAWAY

VICE-PRESIDENTS

Term expires January, 1952:

FRED C. SCOBEY
ALBERT HAERTLEIN

Term expires October, 1952:

WILLIAM R. GLIDDEN
DANIEL V. TERRELL

DIRECTORS

Term expires January, 1952:

WALDO G. BOWMAN
MORRIS GOODKIND
HAROLD L. BLAKESLEE
PAUL L. HOLLAND
EDMUND FRIEDMAN
S. T. HARDING

Term expires January, 1953:

OTTO HOLDEN
FRANK L. WEAVER
GORDON H. BUTLER
LOUIS R. HOWSON
G. BROOKS EARNEST
WALTER J. RYAN
GEORGE W. LAMB

Term expires October, 1952:

MILTON T. WILSON

Term expires October, 1953:

KIRBY SMITH
FRANCIS S. FRIEL
WALLACE L. CHADWICK
NORMAN R. MOORE
BURTON G. DWYRE

PAST-PRESIDENTS

Members of the Board

FRANKLIN THOMAS
ERNEST E. HOWARD

TREASURER

CHARLES E. TROUT

EXECUTIVE SECRETARY

WILLIAM N. CAREY

ASSISTANT TREASURER

GEORGE W. BURPEE

ASSISTANT SECRETARY

E. L. CHANDLER

PROCEEDINGS OF THE SOCIETY

SYDNEY WILMOT
Manager of Technical Publications

HAROLD T. LARSEN
Editor of Technical Publications

COMMITTEE ON PUBLICATIONS

WALDO G. BOWMAN

FRANCIS S. FRIEL
S. T. HARDING
OTTO HOLDEN
LOUIS R. HOWSON
NORMAN R. MOORE